Inference for a reaction-diffusion model of population dynamics in the Neolithic period

Andrew W. Baggaley, Graeme R. Sarson, Anvar Shukurov, Richard J. Boys and Andrew Golightly
Newcastle University, UK
Email: A.W.Baggaley@ncl.ac.uk

Introduction

Without doubt the transition from hunter-gathering to early farming was one of the most important steps made by humanity in moving towards the modern society that we see today. The mechanism of the spread of Neolithic farming, which originated in the Near East around 12,000 years ago remains an important and fascinating question.

Recently there have been a number of studies, applying population dynam- ics models to describe the spread of Neolithic farmers. Whilst some recent work has focused on stoctic methods, the majority of studies have built upon the pioneering work of Ammerman and Cavalli-Sforza [Ammerman, 1973], and sought the solution of a deterministic partial differential equation. The success of this approach has been seen in a number of studies where there is a reasonable agreement between the model and radiocarbon data [Davison, 2006].

In this work we consider a Bayesian statistical approach which allows us to infer plausible values of some of the key parameters in a mathematical model, as it gives this quantity prominence.

Bayesian Inference

Let \( \mu(\theta) \) denote the first time the wavefront arrives at a position \( x \) according to our mathematical model, where \( \theta = (\mu_1, \mu_2, \ldots, \mu_n) \) are the model parameters we wish to make inferences about. We assume the location of the source and the start time of the population movement away from the source are known, though we plan to develop inference methods to determine those parameters in future work. Let \( t_i \) denote the observed arrival time at the radio-carbon site with position \( x_i \), where \( i = 1, \ldots, n \) and \( n \) is the number of radio-carbon sites in our data set. We assume a simple statistical model, with

\[
    t_i = \mu(x_i) + \epsilon_i,
\]

where the \( \epsilon_i \) are independent and identically distributed standard normal random variables.

The Bayesian approach takes information about the unknown model parameters \( \theta \) from the data (through the likelihood function) and combines it with information in an experts prior beliefs. The problem we are studying here is ideally suited to being solved by a Bayesian approach as it provides philosophically sound inferences and computational schemes appropriate for such complex models. These computationally intensive Markov Chain Monte Carlo (MCMC) methods are now routinely used to analyse complex models and data [O’Hagan and West, 2010].

Fig. 4 shows output from the MCMC scheme, including posterior distributions for the model parameters and Fig. 5 gives a comparison between our model and the actual \( ^{14}C \) data at selected sites.

References


Without a doubt the transition from hunter-gathering to early farming was one of the most important steps made by humanity in moving towards the modern society that we see today. The mechanism of the spread of Neolithic farming, which originated in the Near East around 12,000 years ago remains an important and fascinating question.

Recently there have been a number of studies, applying population dynam- ics models to describe the spread of Neolithic farmers. Whilst some recent work has focused on stoctic methods, the majority of studies have built upon the pioneering work of Ammerman and Cavalli-Sforza [Ammerman, 1973], and sought the solution of a deterministic partial differential equation. The success of this approach has been seen in a number of studies where there is a reasonable agreement between the model and radiocarbon data [Davison, 2006].

In this work we consider a Bayesian statistical approach which allows us to infer plausible values of some of the key parameters in a mathematical model, as it gives this quantity prominence.

Bayesian Inference

Let \( \mu(\theta) \) denote the first time the wavefront arrives at a position \( x \) according to our mathematical model, where \( \theta = (\mu_1, \mu_2, \ldots, \mu_n) \) are the model parameters we wish to make inferences about. We assume the location of the source and the start time of the population movement away from the source are known, though we plan to develop inference methods to determine those parameters in future work. Let \( t_i \) denote the observed arrival time at the radio-carbon site with position \( x_i \), where \( i = 1, \ldots, n \) and \( n \) is the number of radio-carbon sites in our data set. We assume a simple statistical model, with

\[
    t_i = \mu(x_i) + \epsilon_i,
\]

where the \( \epsilon_i \) are independent and identically distributed standard normal random variables.

The Bayesian approach takes information about the unknown model parameters \( \theta \) from the data (through the likelihood function) and combines it with information in an experts prior beliefs. The problem we are studying here is ideally suited to being solved by a Bayesian approach as it provides philosophically sound inferences and computational schemes appropriate for such complex models. These computationally intensive Markov Chain Monte Carlo (MCMC) methods are now routinely used to analyse complex models and data [O’Hagan and West, 2010].

Fig. 4 shows output from the MCMC scheme, including posterior distributions for the model parameters and Fig. 5 gives a comparison between our model and the actual \( ^{14}C \) data at selected sites.

References


Without a doubt the transition from hunter-gathering to early farming was one of the most important steps made by humanity in moving towards the modern society that we see today. The mechanism of the spread of Neolithic farming, which originated in the Near East around 12,000 years ago remains an important and fascinating question.

Recently there have been a number of studies, applying population dynam- ics models to describe the spread of Neolithic farmers. Whilst some recent work has focused on stoctic methods, the majority of studies have built upon the pioneering work of Ammerman and Cavalli-Sforza [Ammerman, 1973], and sought the solution of a deterministic partial differential equation. The success of this approach has been seen in a number of studies where there is a reasonable agreement between the model and radiocarbon data [Davison, 2006].

In this work we consider a Bayesian statistical approach which allows us to infer plausible values of some of the key parameters in a mathematical model, as it gives this quantity prominence.

Bayesian Inference

Let \( \mu(\theta) \) denote the first time the wavefront arrives at a position \( x \) according to our mathematical model, where \( \theta = (\mu_1, \mu_2, \ldots, \mu_n) \) are the model parameters we wish to make inferences about. We assume the location of the source and the start time of the population movement away from the source are known, though we plan to develop inference methods to determine those parameters in future work. Let \( t_i \) denote the observed arrival time at the radio-carbon site with position \( x_i \), where \( i = 1, \ldots, n \) and \( n \) is the number of radio-carbon sites in our data set. We assume a simple statistical model, with

\[
    t_i = \mu(x_i) + \epsilon_i,
\]

where the \( \epsilon_i \) are independent and identically distributed standard normal random variables.

The Bayesian approach takes information about the unknown model parameters \( \theta \) from the data (through the likelihood function) and combines it with information in an experts prior beliefs. The problem we are studying here is ideally suited to being solved by a Bayesian approach as it provides philosophically sound inferences and computational schemes appropriate for such complex models. These computationally intensive Markov Chain Monte Carlo (MCMC) methods are now routinely used to analyse complex models and data [O’Hagan and West, 2010].

Fig. 4 shows output from the MCMC scheme, including posterior distributions for the model parameters and Fig. 5 gives a comparison between our model and the actual \( ^{14}C \) data at selected sites.

References