QUESTION 1

Consider a love affair between Romeo and Juliet. Romeo's love for Juliet is represented by $R(t)$ (being love for $R > 0$ and hate for $R < 0$). Similarly, Juliet's love for Romeo is represented by $J(t)$. They respond in kind to each others advances or retractions: the more Romeo loves Juliet, the more she loves him back, and vice versa. The dynamics of their romance is described by the "flow":

$$\vec{v} = (v_R, v_J) = (J, R).$$

The flow is steady so pathlines are streamlines.

(a) Obtain the streamlines $J(R)$. You may leave your answer in an implicit form.

(b) Sketch some typical streamlines in the (R, J) plane. Comment on the fate of their romance.

QUESTION 2

Consider the 2D flow $\vec{v} = (v_x, v_y) = \left(\dfrac{1+t}{x}, 1\right)$ for $t \geq 0$.

(a) Determine $y(x)$ for the streamlines. Sketch some typical streamlines at $t = 0$. Comment on their shape and how they change as t increases.

(b) Obtain the streamlines in parametric form, $x(s), y(s)$. Show that your answer is consistent with that in part (a).

(c) Determine the pathline passing through $(0, 0)$ at $t = 0$ in the form of $x(t)$, $y(t)$ (as implicit functions if necessary).

(d) Determine an expression for the acceleration (vector) of a fluid parcel in the flow.

QUESTION 3

In Example 8 we considered a flow with velocity $\vec{v} = (v_x, v_y) = (\alpha x, -\alpha y)$ and featuring a chemical whose concentration varied in space as $c(x, y, t) = \beta x^2 y e^{-\alpha t}$ (for positive constants α and β). We found that $\dfrac{\mathrm{D}c}{\mathrm{D}t} = 0$.

(a) Obtain the pathlines $x(t), y(t)$ with initial conditions $x(t = 0) = x_0$, $y(t = 0) = y_0$. Hence evaluate the concentration along a pathline, $c\big(x(t), y(t), t\big)$. Is your answer consistent with what you expect?

(b) How does this result change for $\vec{v} = (\alpha x, 0)$ and $c(x, y, t) = \beta x^2 y^2 e^{-\alpha t}$? Is this consistent with $\dfrac{\mathrm{D}c}{\mathrm{D}t}$?